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and the History of General Relativity

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Introduction

JOHN STACHEL

The volume is based on the proceedings of a conference on the history of general relativity held May 8–11, 1986, at the Boston University Conference Center at Osgood Hill (see the preface). As far as we known, this was the first conference exclusively devoted to the subject.

The aim in organizing the conference was to take a first step in remedying a deficiency in current research on the history of twentieth-century physics. For, in spite of the intense and continuing interest in the life and work of Albert Einstein, it is remarkable how little detailed work had been done until very recently on the origins and development of the general theory of relativity. In spite of the fact that Einstein considered this theory the major achievement of his life's work, most research on the history of relativity has concentrated on the special theory. The paucity of research on the general theory is especially striking in comparison to the large and still rapidly growing body of work on the development of the quantum theory. It is interesting to speculate on some possible reasons for the comparative neglect of general relativity.

1. *Technical Difficulty*: Compared to the special theory, work on "internal" aspects of the history of the general theory requires much greater preliminary technical training in several areas of mathematics, most notably differential geometry and tensor calculus. This preliminary hurdle may have served to deter historians of science, and even historically minded physicists who had not previously studied general relativity. It is no accident that the majority of papers in this volume are by physicists who have previously worked on the subject.

2. *Unfashionableness*: From the late 1920s until the late 1950s, general relativity was considered by most physicists a detour well off the main highway of physics, which ran through the quantum theory. (See Jean Eisenstaedt's paper in this volume, "The Low Water Mark of General Relativity, 1925–1955.") Special relativity had to be treated more respectfully, since at least the rudiments of that subject are required to understand relativistic quantum mechanics and quantum field theory or, at a more practical level, to design and operate such devices as klystrons and synchrotrons. By contrast, the

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Inside the Coconut: The Einstein-Cartan Discussion on Distant Parallelism

MICHEL BIEZUNSKI

On February 13, 1930, Albert Einstein wrote to Elie Cartan: "For the moment, this theory seems to me to be like a starved ape who, after a long search, has found an amazing coconut, but cannot open it; so he doesn't even know whether there is anything inside."[1] The theory to which Einstein referred was the theory of "Distant Parallelism," an attempt to unify electromagnetism and gravity. This letter was the twenty-first in a series of thirty-nine that Einstein and Cartan exchanged between May 1929 and May 1932. This correspondence illustrates very clearly what Einstein was seeking when he was working towards a unified field theory, and the nature of the difficulties he encountered in discussion with mathematicians. For while Cartan studied the form of the coconut, Einstein wanted to know what was hidden inside. The failure of their joint work would be due precisely to this difference.

Elie Cartan (1869–1951) was a leading French mathematician who made major contributions to Lie group theory and the theory of spinors. He also worked in the field of systems of partial differential equations and the theory of spaces with linear connections.

The exchange between Einstein and Cartan was an example of those numerous and unsuccessful attempts made by Einstein to unify the theory of gravity and electromagnetism. The letters reveal the special features of the Einsteinian approach.

After developing the general theory of relativity, Einstein tried to reach a further level of generalization by unifying the classical physical interactions: electromagnetism and gravitation. From the early 1920s until the end of his life, he would work in this direction. Einstein proceeded according to a scheme that was characteristic of his attempts to derive a unified field theory: the first step was to find a general geometric space in which there exists a symmetric and an antisymmetric tensor field of rank two. The symmetric one would be identified with the gravitational field, the antisymmetric one with the Maxwell field. The second step was to find the field equations, which would represent the dynamics of the physical interactions. In so doing, he repeated his own approach in developing the general theory of relativity, where he had first generalized Minkowski space to a general Riemannian one, and then after much searching had found the correct field equations (see Stachel 1980, 1986; and Norton 1984). His exchange with Cartan was one of the steps in this program. But Einstein would be mostly concerned with a possible physical interpretation, whereas Cartan would be guided by considerations of logical necessity. It should be noted that this joint work was not Einstein's first attempt to find a mathematical expression for his physical dream of the unification of nature, since he had tried originally to follow Weyl's generalization of Riemannian geometry and later to follow the five-dimensional theory formulated by Kaluza and Klein. And it would not be the last. But let us start the story from the beginning.

Together with the mathematician Roland Weitzenböck, who had worked as early as 1913 on a theory of invariants of physical theories (Weitzenböck 1913–1915),[2] Einstein discovered that there existed a mathematical literature on a type of generalized Riemannian space including both curvature and torsion. Torsion is represented by an antisymmetric tensor that generalizes a classical notion in differential geometry. In those spaces that possess a linear connection, there are two limiting cases: if the torsion is zero, but not the curvature, the space is Riemannian; if the curvature equals zero, but not the torsion, it is a space with "distant parallelism" (*Fernparallelismus*) or "absolute parallelism." The name comes from a remarkable property of a space with torsion and no curvature: one can decide whether or not two vectors are parallel even when they are separated by a finite distance. This is due to the fact that the final direction of a vector is independent of the path followed by the vector undergoing parallel transport. In other words, parallel transport is integrable, and the angle between two vectors has an absolute meaning.

In the "old" theory of gravitation (general theory of relativity) Riemannian space-time had curvature but no torsion. Now Einstein suggested a space with torsion and no curvature. Einstein was led in that direction because there is in this case an antisymmetric tensor of rank two, which could be taken to be the electromagnetic tensor, and another symmetric tensor of rank two, which could be used to represent gravitation. Nothing more was needed for him to hope that there might be a basis here for a unified theory of gravitation and electromagnetism.

In reading an article by Weitzenböck (1928) on the new theory, an article containing a supposedly complete bibliography of mathematical works on the subject of distant parallelism, Cartan was surprised and disappointed not to see his own works mentioned, since he had written on this topic a few years before (see Cartan 1923). Distant parallelism was a particular case of the structure he had studied and called a Euclidean connection. He immediately wrote to Einstein on May 8, 1929 asking him whether he remembered their conversation on this very subject seven years earlier at Hadamard's house in Paris: Cartan had then tried to illustrate what a Riemannian space with *Fernparallelismus* looks like.[3] Cartan had explicitly expounded his ideas to Einstein and it is strange to note that the latter, who was already concerned with attempts to find a unified field theory, had paid no attention to Cartan's explanations. In his answer of May 10, Einstein recognized the debt he owed Cartan and proposed several solutions to repair Weitzenböck's omission, for which he felt himself responsible. These included either a postscript to his own forthcoming article, or a longer piece to be written by Cartan and published independently. Einstein added:

> I didn't at all understand the explanations you gave me in Paris; still less was it clear to me how they might be made useful for physical theory. I first remarked last year that it would be quite natural to add the hypothesis of distant parallelism to the Riemann metric. But only in the last few months have I realized that this actually leads to a theory which corresponds to the hitherto existing knowledge of the physical properties of space, i.e. to a useful set of field equations, almost uniquely determined by formal considerations.[4]

This letter marked the beginning of a very rich correspondence and joint work. The most intense period of the correspondence was between December 1929 and February 1930, during which time Einstein and Cartan exchanged twenty-six letters. It often happened that one or the other wrote several letters a week to his colleague. Several times, Einstein, after sending one letter, did not wait for an answer before sending the next. Cartan was no less eager to succeed than Einstein. Such a contribution would have earned him recognition and broken the isolation he felt in the mathematical community. To have contributed significantly to the new Einsteinian unified theory would have helped enlarge his public.[5]

In his answer to Einstein's letter, Cartan accepted Einstein's proposal to add a historical account of distant parallelism (Cartan 1930) to one of Einstein's articles (Einstein 1930).[6] Einstein told Cartan that he was convinced that he had "found the simplest legitimate characterization of a Riemann metric with distant parallelism that can occur in physics."[7] He referred to a system of twenty-two field equations built out of the torsion tensor along with sixteen complementary equations also including this tensor.

But the first difficulty that Einstein and Cartan would encounter concerned the choice of the appropriate set of equations. And the quests of Cartan, a mathematician, and that of Einstein, a physicist, were of different natures. Cartan was concerned with the logical and formal aspects of the theory, Einstein, with its physical interpretation.

Cartan explained in a letter of December 3, 1929 that he had sought but could not find a decisive argument to establish that Einstein's choice of equations was privileged among others, as he had done for the general theory of relativity (Cartan 1922).[8] Furthermore, he was astonished that Einstein had managed to find this particular system.

Cartan inserted with this letter a note in which he defined the concepts he used, concepts that would be the topic of the bulk of his correspondence with Einstein. He introduced, among other things, a measure of the generality of a system of partial differential equations, which he called the "generality index," equal to the number of arbitrary functions on which the general solution of a deterministic system depends. He also explained what he meant by "systems in involution": A system of linear partial differential equations is said to be in involution "if any 0-dimensional solution is part of at least one 1-dimensional solution, if any *arbitrary* 1-dimensional solution is part of at least one 2-dimensional solution, and so on."[9]

Cartan proposed one alternative system of twenty-two field equations, and another one with fifteen equations, which he himself preferred because it gave rise to a richer geometrical scheme. In the same letter he wrote:

> When thinking it over, I believe that the degree of generality of the geometrical scheme corresponding to your system of 22 equations is a bit weak, the old classical theories of gravitation and electromagnetism give to physics a greater degree of generality.

This twenty-two–equation system had a generality index of 12. The fifteen-equation system had a generality index of 18. That is why the second system is richer, Cartan explained.

Einstein answered on December 8 that he disagreed even about the number of equations. This shows how far he was from being convinced by Cartan's reasoning, which was based on a fixed mathematical structure, the number of whose equations could not be modified without a global change of the system. Moreover, Einstein preferred a system with a lower generality index:

> In my opinion, a theory is the more valuable the more strongly it restricts possibilities, without coming into conflict with reality. It is like a wanted poster which is supposed to characterize a criminal; the *more precisely* it points him out the better.[10]

In his next letter, however, Einstein recognized that he had not fully understood Cartan's explanation of the generality index and urged him again to answer the question whether his own system of equations was especially privileged from the point of view of their generality index.[11] Cartan answered that it is impossible to tell whether this system is privileged in any way. But, he added: "*there is no system* MORE *determined* than yours."

Cartan then raised a question that showed his desire to find a physical concomitant for his generality index:

> Is any solution of Maxwell's equations which is defined only in a small region of space-time, *physically* admissible? To put it another way, let us assume that for [a finite region of space-time] we have functions defining the electromagnetic field in this domain and satisfying Maxwell's equations *in this limited domain*; do you think that such a field can actually exist? The local state is certainly influenced by what happens elsewhere, outside the domain under consideration, but can we deal with the possibilities, infinite in number, that exist outside the domain so as to determine any *local* solution of Maxwell's equations inside the domain?
>
> This question seems important because, if the answer is affirmative, it would give a *physical* meaning and not merely a mathematical one, to the generality index of the system of partial differential equations that determines the field.[12]

Indeed, Einstein was looking for mathematical approval. He had already enjoyed this after the publication of the general theory of relativity. Einstein wanted his mathematical equations to be the only ones. Cartan, on the other hand, was looking for a physical meaning, perhaps to prove to himself that his theory could have some utility to Einstein and was not simply a mathematical exercise. But in reality, it soon became clear that, whereas Cartan's arguments were almost always based on formal considerations, Einstein's sensitivity to problems of physical interpretation led him to stronger restrictions than those imposed by a simple quest for some physical meaning. For example, Einstein objected to Cartan's system that in the latter:

> The g_{ik} alone have an autonomous causality, independent of the parallel structure. This can be expressed as follows: the electromagnetic field has no reaction upon the gravitational field. This completely contradicts physical expectations.[13]

Einstein emphasized that physics requires the field to be free of singularities everywhere. This condition dramatically limits the choice of a set of equations. He raised the question of the existence of nontrivial solutions whose generality index is equal to zero. These systems, like those of classical mechanics, would not be fixed "by an arbitrary choice of given *functions* but by a choice of parameters (numbers). For some time I was convinced that the true laws of nature would have to be of such a kind."[14] But, he added,

> It might also be possible that that high degree of constraint (which I have no doubt is realized in the true laws of nature) is based on something else. The indicated small measure of arbitrariness (in Nature) could also be grounded in the requirement that singularities are to be excluded from all space!

To that question, Cartan answered that for any given torsion there exists a space having that torsion in which the solution of the field equations is singularity free; and these singularity-free solutions correspond to algebraic solutions of the structure equations of the associated Lie group.[15] But Cartan admitted that this is only a particular case, and that he did not know any method allowing one to deal with this problem in general.

Einstein had quite a different point of view on the matter. In a letter of January 7, 1930, he began to distance himself from Cartan's approach, in an elegant way and with humor: "I must further apologize for the thoughtlessness of my questions. There's a lovely old saying: A fool can ask more questions than a wise man can answer."[16]

He added that he was happy that Cartan felt confidence in the fact that there exists no involutive system with a lower degree of freedom. Einstein had tried himself to find one in vain. But he had moved to a concern with the physically acceptable solutions of the field equations. He explained again to Cartan his requirement that the solutions be singularity-free. If they were not, there would exist uncharged point-masses at rest with respect to each other, which "contradicts experience," first because these particles would not be subject to gravitation, and second because they would have no charge. It is interesting to note that, two years before the discovery of the neutron, Einstein had no intuition of the existence of such uncharged particles.

From this point on, the widening of the gap between the two approaches becomes more apparent.[12] The numerous letters in the period between January and February 1930 saw both protagonists go their own way. The details of these letters are less interesting, from a comparative point of view, because they are like two parallel monologues. For example, Einstein closed his letter of January 21, 1930, with this sentence: "Thus you must have *still more* patience and compassion for a poor physicist whose destiny has led him to these troubled pastures."[18]

A month later, Einstein had not gone any further. He was still interested in the existence of singularity-free solutions that could represent electrons and protons, "for without the solution of this difficult problem I feel no judgment can be passed on the usefulness of the theory."[19] It was in this letter that he introduced the metaphor of the coconut quoted at the beginning of this paper. Cartan answered on February 17:

> (The whole problem is to find a singularity-free solution general enough to be physically interpretable. But are we really sure that such solutions exist and that the coconut contains something inside?)
>
> We find ourselves in front of a wall and we mathematicians are quite at a loss as to how make a hole in it. One can only hope for some miracle of divination, but then you already have had several.[20]

But the miracle didn't happen.

Five months passed before Einstein got round to answering this letter. During that period, Cartan continued to pursue his research in this domain. He published a popular article on the new theory (Cartan 1931a) and announced to Einstein that he was just finishing another article on systems in involution and the theory of relativity that he would send on (Cartan 1931b). Einstein wrote that, in the meantime, he had been working with his new collaborator, Walter Mayer, and had decided to abandon the previous field equations:

> The reason is that it appears that, according to those field equations there are no gravitational effects, since static solutions with arbitrarily many point-masses (singularities) exist for which only h_{44} is non-zero....[21]

Einstein was trying, together with Mayer, to find new field equations using another approach. The reason for rejecting the solution was essentially the presence of singularities (Einstein and Meyer 1930). This prompted Cartan to suggest once again his fifteen-equation system and no doubt to feel that his work could serve more than ever as a mathematical basis for Einstein's research. But Einstein waited nearly nine months before replying. On his way to the United States on board the steamer *The San Francisco*, he wrote to thank Cartan for the promised article on involution theory in these terms: "I have read with great enjoyment your work on systems in involution. This seems to me a truly important contribution to the theory of partial differential equations."[22] As the paper was entitled "On the Theory of Systems in Involution *and its Applications to the Theory of Relativity*" (my emphasis), this sentence must have come as a shock to Cartan, because of its implied rejection of the usefulness of his work in the field of relativity and unified field theory. And the rest of the letter left no doubt:

> In any case, I have now completely given up the method of distant parallelism. It seems that this structure has nothing to do with the true character of space. For some years, together with Dr. Mayer, I have pursued another theory, that of the 5-vector and 5-metric in a four dimensional metric continuum. This theory not only yields Maxwell's equations in a natural way but also—as I have recently discovered—an extension of them which admits continuously distributed charged masses. I have hopes that this theory really comes closer to the structure of physical space without its basic laws having to be given a merely statistical interpretation. I absolutely cannot reconcile myself to this dogma of the new generation of physicists, no matter how enticing the arguments are that are made for it....
>
> The main reason for the uselessness of the distant parallelism construction lies, I feel, in that one can attribute absolutely no physical meaning to the "straight lines" of the theory, while the physically meaningful (macroscopic) equations of motion cannot be obtained from it. In other words, the $h_{s\nu}$ give rise to no useful representation of the electromagnetic field.[23]

The few remaining letters speak only of questions touching purely mathematical matters concerning systems of differential equations.

It is a generally accepted view that during the last thirty years of his life Einstein was constantly changing theories and enthusiasms in the search for a unified field theory. But this work with Cartan reveals there was an underlying unity, that he was using the same methods and the same concepts that he used to derive his general theory of relativity, and he was trying to repeat once again his earlier success. He wanted a theory that is complete, general, aesthetically satisfying, and unified. The means were less important in his eyes. He used a mathematical theory as a tool, and never more. He was concerned with physical problems, and refused to be caught in the trap of formal, mathematical structures.[24]

Even if, as the Einstein-Cartan correspondence shows, these two men followed from the start parallel but distant paths, which never really met, the appreciation that each felt for the other was genuine. As Cartan said in his article popularizing Einstein's distant parallelism theory:

> We see ... the variety of points of view from which the unified field theory can be regarded and also the difficulty of the problems that it raises. But Mr. Einstein is not someone who is frightened by difficulties, and even if his attempt is not ultimately successful, it will have to force us to reflect on the deep questions which lie at the heart of science.[25]

Acknowledgments. I would like to thank Jim Ritter, who translated Einstein's correspondence with Cartan into English, for his scientific and linguistic help.

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The Einstein–de Sitter Controversy of 1916–1917 and the Rise of Relativistic Cosmology

Pierre Kerszberg

1. Introduction

In textbooks of modern cosmology, and in histories of the subject, the name of Willem de Sitter occurs in relatively few contexts. He is standardly associated with the discovery, in the year 1917, of a very strange and unexpected solution of Einstein's field equations: an *empty* universe need not have the metric of Minkowski flat space-time (see, for instance, Peebles 1980a, p. 4; Whitrow 1980, p. 284). This result is itself a reaction to Einstein's announcement, earlier in the same year, of a metric for the first relativistic model of the whole universe—the so-called cylindrical universe (Einstein 1917). Current literature also mentions an Einstein–de Sitter model of the universe, which has nothing to do with either of these solutions. It refers to a model that de Sitter constructed together with Einstein some fifteen years later, when the rediscovery of nonstatic solutions made it clear that the static solutions of 1917 were particular cases of a much wider class of solutions. In fact, the Einstein–de Sitter model is known for yielding the simplest of all nonstatic cases: its geometry is an expanding Euclidean space, relieved of complicated relations among available variables. This simplicity has made it appropriate to the study of the meaning of these more complicated relations, such as the occurrence of terms representing the pressure.

Historical studies of contemporary cosmology also draw attention to the fact that the role of de Sitter was not restricted to these two contributions. A very decisive role was played by de Sitter's early critique of some of the most fundamental philosophical arguments Einstein had put forward when he constructed the theory of general relativity, prior to the emergence of cosmology. These studies suggest, indeed, that Einstein developed his cosmological ideas as a response to de Sitter, who had expressed objections to Einstein's views on the problem of the relativity of rotation and the origin of inertia. Thus, after having commented on Einstein's first cosmological memoir, Jacques Merleau-Ponty wrote: "It was, in part, to take up de Sitter's challenge that Einstein had sought with so much obstinacy to find solutions to the field equations that would be compatible with the principle of the relativity of